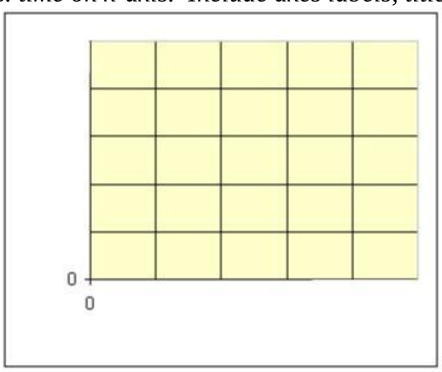
Mass Balance for a water bucket

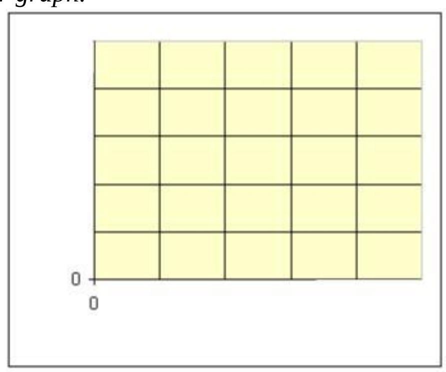
Gotohttp://www.atmosedu.com/physlets/GlobalPollution/WaterBucket.htm
and then follow the online instructions.
A simple bucket with 2.0 gal/min flow into the bucket.

time(min)	Content(gallons)
0.0	0.0
1.0	2.0
2.0	4.0
3.0	
4.0	
5.0	
6.0	
7.0	
8.0	
9.0	
10.0	

Fill in the table above and then use the left set of axes below to make a graph of bucket content on y-axis vs. time on x-axis. Include axes labels, titles, and units on your graph.



Top table



Bottom table

A leaky bucket (lifetime = 10 min)

time(min)	Content(gallons)
0.0	10.0
1.0	9.0
2.0	8.1
3.0	
4.0	
5.0	
6.0	
7.0	
8.0	
9.0	
10.0	

Fill in the table above and then use the axes on the right above to make a graph of bucket content on y-axis vs. time on x-axis. Include axes labels, titles, and units on your graph.

Using the online Model for a simple bucket

Go ***http://www.atmosedu.com/physlets/GlobalPollution/WaterB2.htm***

Objective: Use the online model to explore how a bucket that does not leak fills up over time and to determine an equation that describes this process.

Start with an empty bucket (content, Co=0) and no leak (life-time is VERY large ~10000000 minutes). This is the simple bucket.

Q1: Using Co=0, life-time=10000000 minutes, and S=2 gal/min.
How much water is in the bucket in 50 min?

Q2: repeat Q1 for S=1.0 gal/min, S=0.5 gal/min, and S=0.25gal/min. You can click the mouse down and move around the graph to read values right off the graph.

S(gal/min)	C (at 50 mn)
0.25	
0.5	
1.0	
2.0	

Q3: For the four flow rates {S=0.25gal/min, 0.5gal/min, 1.0gal/min, and 2.0 gal/min} rank them in order of water content at 50 min from greatest to least.

greatest			least

Q4: If the flow rate is S=5.0 gal/min and Co=0.0, how much water is in the tank in 4.0 min?

Q5: If the flow rate is S=5.0 gal/min and Co=20.0 gallons, how much water is in the tank in 4.0 min?

Q6: For the four pairs A=(3.0,10), B=(5.0,0.0), C=(2.0, 30), and D=(0.0,35)(flow rate in gal/min, initial content in gallons) rank them in order of greatest to least water content at 4.0 min.

greatest			least

Write an equation for the content C at any time t that involves the initial water content (Co), the flow rate into the bucket (S), and time (t)? Try it and check to make sure it works. Write your equation below.

C=

Using the online Model for a leaky bucket with S=constant

http://cs.clark.edu/~mac/physlets/GlobalPollution/WaterB2.htm

Objective: Explore how the equilibrium water level depends on the initial water level for a leaky bucket.

Q7: Start with Co=0, lifetime=10 min, and S=4gal/min. What is the final equilibrium water content?

Q8: Repeat Q7 except use Co=20gal, 40gal, 60gal, & 100gal. This would be a good time to use the four different run buttons. (fill in the table below for your answer)

Co (gal)	Ceq
0	
20	
40	
60	
100	

Q9: Does the equilibrium content depend on the initial content?

Objective: Explore how the equilibrium water content depends on the lifetime for a leaky bucket. (here we keep the flow rate into the bucket fixed at 4 gal/min)

Q10: Using Co=0 and S=4gal/min and a lifetime of 2 min what is the final equilibrium water content. Repeat using lifetimes of 5, 10, 20, and 25 minutes.

lifetime (min)	Ceq
2	
5	
10	
20	
25	

Q11: Does the bucket equilibrium content depend on the lifetime?

How does doubling the lifetime from 5 to 10 minutes influence the equilibrium content?

How does quadrupling the lifetime from 5 to 20 minutes influence the equilibrium content?

Q12: For bucket lifetimes of 10, 20, 50, & 100 minutes, Rank each bucket from most leaky to least leaky.

most leaky			least leaky

Q13: For bucket life-times of 10, 20, 50, & 100 minutes, Rank each bucket from highest to lowest equilibrium content.

Highest equilibrium			lowest equilibrium

Objective: Explore how the equilibrium water content depends on the flow rate into a leaky bucket (Source, S) for a leaky bucket. (here we keep the lifetime fixed at 10 min)

Q14: Find the equilibrium water content using Co=0, lifetime=10 minutes, and S=2 gal/min. Repeat this experiment using S = 4, 6, 8, and 10 gallons/minute. { keep Co=0, lifetime=10 min }

S (gal/min)	Ceq
2	
4	
6	
8	
10	

Q15: Does the bucket equilibrium content depend on the flow source (S)?

How does doubling S from 2 to 4 gal/min influence the equilibrium content?

How does quadrupling S from 2 to 8 gal/min influence the equilibrium content?

Q16: For S =2 , 4, 6, and 8 gallons/minute, Rank each from highest to lowest equilibrium content.

Highest equilibrium			lowest equilibrium

Q17: Which of these equations best describes the equilibrium content in a bucket with an inflow S and a given lifetime?
a. Ceq=S*(lifetime)
b. Ceq=(lifetime)/S
c. Ceq=S/(lifetime)
d. Ceq=S+(Lifetime)

More fun with this Generic Model.

This water bucket model structure can be used for several other situations that relate to your everyday life. Here are two examples. Work through both of these examples to complete the assignment.

Example 1. Energy loss from your home. It is freezing cold outside (0.0 ^oC) and you are using a heater with heater strength S. S in our model represents the energy flow into your home in units of ^oC/hr and the temperature (T in ^oC) is directly related to the energy content in your home. Instead of using C for content we will use temperature T as a measure of the energy content.

For a typical home the lifetime (or residence time) of energy in your home is about 4.0 hours (better insulation will increase this time and poorer insulation or more leaks will decrease this time). For a normal sized home with air, walls, and furniture that retain heat it is not unrealistic to set the value of S equal to the numerical value of the heater power in kilo-Watts. [S =1.0 ^oC/(kWatts-hr) (heater power in kWatts)]. The equation for temperature change per hour (ΔT) for our home becomes:

$$\Delta T = S - \frac{T}{4.0}$$

Here **S** has the numerical value of the kilo-Watt rating of your heater and **T** is much the inside temperature is above the outside (zero). Use the model to think about our model home instead of a bucket.

Q1: With **S= 5 kWatts** what is the equilibrium temperature of the home? (run the model if you need to but the equation you came up with in question 17 above works well)

T_{eq} =______________________

If we insulate this home better to cut heat loss by half, the residence time goes from 4.0 hours to 8.0 hours.

Q2: For this situation what is the equilibrium temperature for a 2 kilo-Watt heater?

T_{eq} =______________________

Actually the T in our equation is the temperature inside relative to that outside. When the outside temperature is 0.0^{o}C (32^{o}F), then T is just the inside temperature.

$$T = T_{inside} - T_{outside} \qquad \text{or} \qquad T_{inside} = T + T_{outside}$$

For example, when our model predicts an equilibrium temperature of 20^{o}C (68^{o}F) and the outside temperature is 5^{o}C (41^{o}F), then the inside temperature at equilibrium is 25^{o}C (77^{o}F), OR if our model predicts an equilibrium temperature of 20^{o}C (68^{o}F) and the outside temperature is -5^{o}C (23^{o}F), then the inside temperature at equilibrium is 15^{o}C (59^{o}F)

Q3: It is -10^{o}C (14^{o}F) outside and you want the home to have a room temperature of 20^{o}C (68^{o}F). Assuming a residence time of 4 hours:

What is the equilibrium temperature needed (T_{inside}-$T_{outside}$)?

What heater power do you need?

Up to this point you have not actually needed to run the model on the computer because your equation relating equilibrium level to source and residence time could be used. In this next question it is easiest to run the model.

Q4: The heater has been off for several days so the inside and outside temperatures are both -10^{o}C (14^{o}F). You turn on **three** 2 kilo-Watt heaters at 6 PM. **Use the model** to estimate how warm the home will be by 9 PM. Assume a residence time of 4 hours. Remember the model will give you the value of (T_{inside}-$T_{outside}$).

Example 2. Paying off your loan.

The water bucket model structure also works for a loan balance (**B**) as the content, but if we want to use the online model for this problem we will have to be careful with + / - signs of the different values. For loan balances, S must be negative (as model input) to represent your monthly payment (P) reducing your loan balance, and the lifetime is also negative to represent the idea of interest being **added** to your balance.

If the interest rate is 0.5 %/month {(6%/year)/(12 months/year) } the equation for the change in loan balance, B, during 1 month is:

$$\Delta B = -P + (0.005)B = -P + \frac{B}{200}.$$

Contrast this with the water bucket equation $\Delta C = S - \dfrac{C}{(\text{residence time})}$.

By comparing these equations carefully we can see why, if we want to use the same water bucket model structure, we must use –Payment (P) instead of S and +Balance/(residence time) instead of –C/(residence time)

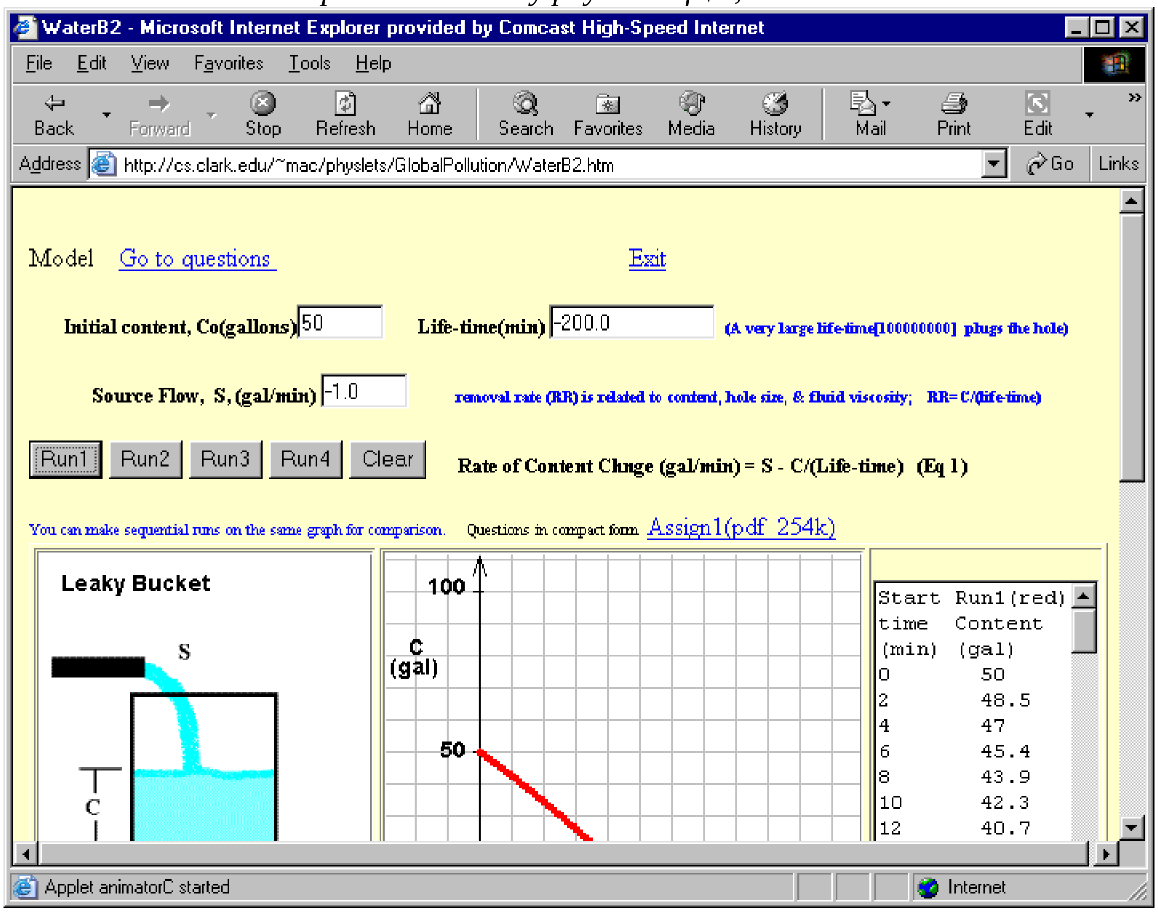
Notice that the "residence time" in months that you actually use in the model is: –(the inverse of monthly interest rate).

Residence time = $-\dfrac{1}{0.005} = -200$ months.

You buy a new car that costs 50,000 dollars.

Q1: How long will it take to pay off a loan at 6% if your monthly payment is 1200 per month?

To use the water bucket model structure for this use units of $1000. For this case you set the initial content to 50, the source to –1.2, and the lifetime to –200. The screen below shows a similar run except with a monthly payment of $1,000.



Q2: What payment do you need to pay the $50,000 car loan off in 5 years? Do this by trial and error and scroll through the output table or carefully read the graph.

Q3: If you buy a house for $100,000 at 6% interest, what payment is needed to pay it off in 8 years?

Q4: If you buy a house for $100,000 at 4% interest, what payment is needed to pay it off in 8 years?

Simple savings account. Describe how the water bucket model structure can be used for a simple retirement type savings account.
That is:

Q5: will P be positive or negative?

Q6: will the residence time be positive or negative?

Hint: Both your monthly payment (deposit) and interest earned add to you account. One of these is like the water bucket and one of them is the opposite. The correct answer is that one of them is positive and one is negative.

Assume that you can get 6% interest on your savings, you start with $10,000, and that you deposit $300.00 per month.

Q7: How much is in your account in 10 years?

Q8: How much would you have to deposit monthly to have 100,000 in 10 years?

Q9: What interest rate would you have to earn to have $100,000 in 10 years if you deposited $300.00 each month?

Appendix: This Appendix may be useful to students with a strong mathematical background. The solution to the water bucket model equation for constant source **S,** initial content $\mathbf{C_o}$**,** and residence time $\boldsymbol{\tau}$

$$\frac{dC}{dt} = S - \frac{C}{\tau}$$

is

$$C = S\tau\left(1 - e^{-t/\tau}\right) + C_0 e^{-t/\tau}$$

Notice as t grows very large the exponential terms go to zero and **C** approaches $\mathbf{S\tau}$ (Ceq). You can use this solution to answer the questions for **Example 1. Energy loss from your home.** or **Example 2. Paying off your loan.** Using the online model is probably easiest.